OPTOELECTRONICS LETTERS Vol.19 No.5, 15 May 2023

Implementation of quantum optical tristate CNOT gate using frequency encoding principle with a semiconductor optical amplifier

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(Received 15 November 2022; Revised 24 January 2023)

Controlled NOT (CNOT) gate is well known because of its several advantages in quantum computing and information processing. In the area of quantum computing, several methods of CNOT gates were established in last few years. In this paper, we propose a new approach of implementation of tristate CNOT operation with light as information carrying signal. To do this, the frequency encoding method has been exploited for successful realization of the CNOT gate with light.

Document code: A **Article ID:** 1673-1905(2023)05-0269-5

DOI https://doi.org/10.1007/s11801-023-2195-x

Quantum optical logic gates are the key components of a quantum optical computer. Quantum computation deals with qubits or quantum bits which are treated as the quantum analogue of classical bits and can be used to store quantum information[1]. Different types of encoding processes, such as phase encoding, frequency encoding, polarization encoding, intensity encoding, hybrid encoding, etc are used for developing the qubits[2]. Optics is already regarded as a powerful and promising candidate in quantum computation, entanglement, communication and data processing because of its inherent nature of creating quantum states. Quantum optical logic systems are highly useful in superfast optical logic operation, data encryption, secured and noise free communication[1-3]. Recently, semiconductor optical amplifier (SOA) is regarded as a promising optical switch for its compactness, large optical gain, large bandwidth, high stability, fast response time, low power consumption, ease of fabrication in optical network and ability to show different types nonlinearities such as cross gain modulation (XGM), cross phase modulation (XPM), self-gain modulation (SGM), self-phase modulation (SPM), cross polarization modulation (XpolM), four wave mixing (FWM), etc[4,5]. Depending upon such nonlinearities, the device can be used in optical signal processing, fast optical switching, frequency conversion, signal amplification, etc[5,6]. Over last few decades, several well-known quantum optical logic gates and their integrated counterparts were implemented using different techniques, where qubits are tactfully managed to do the logic operations[7-14]. A novel approach of implementing all-optical Pauli Y gate using phase and polarization encoding of light was proposed by DEY et al[8]. All-optical scheme for developing integrated Pauli X, Y and Z gates was proposed, where electro-optic Pockels material was used as an optical switch[9,10]. Quantum optical tristate Pauli X, Y, Z gates have also been proposed, where phase encoding technique has been used to encode the qubits[11]. Implementation of quantum optical intensity encoded Fredkin and Toffoli gate using Kerr type nonlinear material have been also reported[12-14]. Implementation of quantum Fredkin gate and Swap gate using linear optical elements has been suggested by ZHU et al[15]. An implementation of Toffoli gate with superconducting circuits has been proposed by FEDOROV et al[16]. Design and realization of quantum optical two state CNOT gate has been proposed by many researchers[17-22]. LOPES et al[19] designed a linear-optical CNOT gate for two qubit photons where polarization and momentum of photon act as control and target qubits respectively. An optical implementation of quantum CNOT gate using polarization encoded qubits has also been reported[20,21]. Implementation of quantum optical two state CNOT gate has been also proposed where switching operations were done by Kerr type nonlinear material[22]. Square root of various quantum logic gates such as square root of Pauli gates and controlled Z gate have also been proposed[23-26]. In this paper, an all-optical scheme for realization of frequency encoded quantum optical tristate CNOT gate is proposed. A proper truth table and gate matrix for quantum optical tristate CNOT gate have been designed. The SOA based optical switches, such as frequency converters and add/drop multiplexers, are used to develop the whole system. Since the qubits are formed by using

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frequency encoding principle, the system ensures very high bit rate, very low bit error rate and very high signal to noise ratio. Also, data handling capacity and speed of different types of optical digital, arithmetic and algebraic operations can be improved significantly due to the application of tristate logic. The prime focus of this work is using tristate logic for implementation of the CNOT operation with light. CNOT gates in two qubits are already proposed by different researchers, but the use of tristate based two qubit CNOT gate hasn't been proposed. Here, the use of tristate logic accommodates more number of information for processing in comparison to two state systems. That is why this proposed system is more flexible and advantageous than the two state CNOT gate. The use of frequency encoding principle and tristate logic system is the main point of attention here, which can make the system more practical.

CNOT gate is one of the most essential logic gates in the logic family which is widely used to perform information processing task in quantum optical logic systems. It is a two-qubit reversible logic gate where the first qubit is referred as the control qubit and the second qubit is referred as the target qubit. In tristate logic, information is represented by three distinct states 0, 1 and $\bar{1}$. The proposed truth table of tristate CNOT gate is depicted in Tab.1[27]. Here, the operation of tristate two qubit CNOT gate is supported. It is seen that the output O_1 is the straightforward output, the same as the input state A, whereas the output O_2 is changed accordingly to the input state B. When the input state A is 1, the state B=1 changes to the state $O_2=\bar{1}$, the state $B=\bar{1}$ changes to the state $O_2=1$ and the state B=0 remains unchanged. When the input state A is $\bar{1}$, the state B=0 changes to the state $O_2=1$, the state B=1 changes to $O_2=0$ and the state $B=\bar{1}$ remains unchanged.

Tab.1 Truth table of tristate CNOT gate

Inputs		Outputs	
A	B	O_1	O_2
0	0	0	0
0	1	0	1
0	$\bar{1}$	0	$\bar{1}$
1	0	1	0
1	1	1	$\bar{1}$
1	$\bar{1}$	1	1
$\bar{1}$	0	$\bar{1}$	1
$\bar{1}$	1	$\bar{1}$	0
$\bar{1}$	$\bar{1}$	$\bar{1}$	$\bar{1}$

The gate matrix of quantum optical tristate CNOT gate is represented by a 9×9 square matrix as $\begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & \cdots & m_{1,8} & m_{1,9} \\ m_{2,1} & m_{2,2} & \cdots & \cdots & m_{2,8} & m_{2,9} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{8,1} & m_{8,2} & \cdots & \cdots & m_{8,8} & m_{8,9} \\ m_{9,1} & m_{9,2} & \cdots & \cdots & m_{9,8} & m_{9,9} \end{bmatrix}_{9\times 9}$.

The state of light can be represented by a 9×1 column matrix as $\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ C_8 \\ C_9 \end{bmatrix}_{9\times 1}$.

In tristate logic systems, the states $|0\rangle$, $|1\rangle$, $|\bar{1}\rangle$ can be represented by 3×1 column matrices like $|0\rangle=\begin{bmatrix}1\\0\\0\end{bmatrix}$, $|1\rangle=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $|\bar{1}\rangle=\begin{bmatrix}0\\1\\0\end{bmatrix}$, respectively.

From the truth table, one can notice that when the control qubit is in $|0\rangle$ state, the target qubit remains in the same state and when the control qubit is in $|1\rangle$ or $|\bar{1}\rangle$ state, the target qubit changes by following a tristate NOT operation.

Now, when the control qubit is in $|0\rangle$ state, tristate CNOT gate transforms the basis states such that the target qubit remains in the same state.

$$|00\rangle \xrightarrow{\text{tristate CNOT}} |00\rangle, \tag{1}$$

$$|01\rangle \xrightarrow{\text{tristate CNOT}} |01\rangle, \tag{2}$$

$$|0\bar{1}\rangle \xrightarrow{\text{tristate CNOT}} |0\bar{1}\rangle. \tag{3}$$

For control qubit in $|1\rangle$ state, the target qubit changes state by favouring a tristate NOT operation under the action of tristate CNOT gate and the basis states are transformed as shown in the following operations.

$$|10\rangle \xrightarrow{\text{tristate CNOT}} |10\rangle, \tag{4}$$

$$|11\rangle \xrightarrow{\text{tristate CNOT}} |1\bar{1}\rangle, \tag{5}$$

$$|1\bar{1}\rangle \xrightarrow{\text{tristate CNOT}} |11\rangle. \tag{6}$$

Again, when the control qubit is in $|\bar{1}\rangle$ state, tristate CNOT gate transforms the basis states such that the target qubit changes by favouring a tristate NOT operation.

$$|\bar{1}0\rangle \xrightarrow{\text{tristate CNOT}} |\bar{1}1\rangle, \tag{7}$$

$$|\bar{1}1\rangle \xrightarrow{\text{tristate CNOT}} |\bar{1}0\rangle, \tag{8}$$

$$|\bar{1}\bar{1}\rangle \xrightarrow{\text{tristate CNOT}} |\bar{1}\bar{1}\rangle. \tag{9}$$

Now expressing the operations Eq.(1) to Eq.(9) in matrix form and then solving all the matrix equations one can have $m_{1,1}=1$, $m_{2,2}=1$, $m_{3,3}=1$, $m_{4,6}=1$, $m_{5,5}=1$, $m_{6,4}=1$, $m_{7,7}=1$, $m_{8,9}=1$, $m_{9,8}=1$, and all other elements are zero.

Finally, the gate matrix of quantum optical tristate CNOT gate becomes $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{9\times 9}$.

The developed gate matrix for quantum optical tristate CNOT gate is unitary and reversible in nature, which is very much essential for a quantum operation[27].

An all-optical scheme for realization of quantum optical tristate CNOT gate using frequency encoding technique is shown in Fig.1. SOA based optical switches, such as frequency converters and add/drop multiplexers, mirrors, optical pass filters, etc, are used to develop the entire system. Here, FCs are frequency converters; F_1 is f_1 pass filter, F_2 is f_2 pass filter, F_3 is f_3 pass filter; A_1, A_2, A_3,, A_4 are add/drop multiplexers; Cs are circulators; dotted lines represent probe beams. A and B are the input optical channels that may have either f_1 or f_2 or f_3 frequency of light. O_1 and O_2 are output optical channels. Logic state '0' is encoded by f_1 frequency of light signal, '1' state by f_2 and '$\bar{1}$' state by f_3 frequency of light signal.

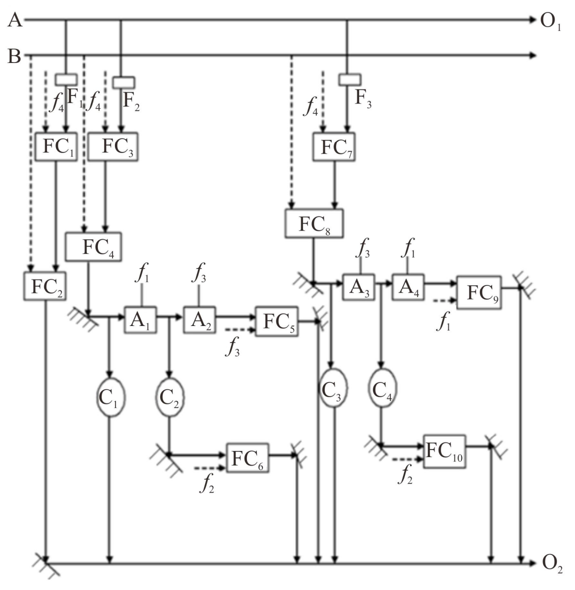

Fig.1 Block diagram of tristate CNOT gate

The design of the circuit with elements FCs, Cs, As and Fs is done in such a way that the required operation order is maintained. Output frequency O_1 remains the same as that of the input frequency A and output frequency O_2 changes according to the input frequency B. If A takes the frequency f_1, output O_2 remains the same as that of B. When A takes f_2 frequency and B takes f_1, output O_2 also remains the same as that of B. Again, when f_2 frequency is applied at both the input channels A and B, then frequency applied at B changes to f_3 and appears at O_2. When A takes f_2 frequency and B takes f_3 frequency, then one can get f_2, frequency at the output O_2. Now, when f_3 frequency is applied at A and f_1 at B, then f_2 frequency is obtained from the output O_2. When A takes f_3 frequency and B takes f_2, then the frequency applied at B changes to f_1 and appears at O_2. When f_3 frequency is applied at both the input channels A and B, output O_2 remains the same as that of B. The detail operation of the system is now described below.

For all cases, output O_1 is taken from the input optical channel A.

In the first case, when A=f_1, B=f_1=f_2=f_3, f_1 frequency of light signal from input optical channel A passes through the f_1 pass filter F_1 and is applied as a strong pump beam to the FC_1. A constant probe beam of f_4 frequency is present at the input of FC_1. So, f_4 frequency of light signal is obtained from the output of FC_1 and it acts as a strong pump beam of FC_2. Now, light from input optical channel B is applied as a weak probe beam to the FC_2. When B=f_1, then f_1 frequency is obtained from the output of FC_2 which is transferred to the output optical channel O_2. Remaining portion of the system does not work due to absence of pump beam. Similarly, when B=f_2, then O_2=f_2 and when B=f_3, we obtain O_2=f_3.

When A=f_2, B=f_1=f_2=f_3, then f_2 frequency of light from input optical channel A acts as a strong pump beam to the FC_3 after passing through the f_2 pass filter F_2. A weak probe beam of f_4 frequency is applied to the FC_3. As a result, f_4 frequency is obtained from the output of FC_3 and acts as a strong pump beam to the FC_4. Light from input optical channel B acts as a weak probe beam to the FC_4. When B=f_1, then f_1 frequency is obtained from the output of FC_4 and this frequency of light is reflected by the add/drop multiplexer A_1 as it is tuned at f_1 frequency and is collected by circulator C_1 and appears at the output optical channel O_2. Rest of the circuit cannot operate due to absence of pump beam. A_2 is tuned at a frequency f_3. When B=f_2, then f_2 frequency is obtained from the output of FC_4 and it is passed by A_1 and A_2 and behaves as a strong pump beam to the FC_5. A constant probe beam of f_3 frequency is present at the input of FC_5. So, at the output of FC_5, f_3 frequency is obtained which appears at the output optical channel O_2. When B=f_3, then f_3 frequency of light is obtained from the output of FC_4. A_1 passes this frequency but A_2 reflects it and it is then collected by the circulator C_2 and is applied as a strong pump beam to the FC_6. A weak probe beam of f_2 frequency is present at the input of FC_6. So, f_2 frequency of light is obtained from the output of FC_6 and it appears at the output optical channel O_2.

When A=f_3, B=f_1=f_2=f_3, then f_3 frequency of light from input optical channel A passes through the f_3 pass filter F_3 and it acts as a strong pump beam to the FC_7. A weak probe beam of f_4 frequency is present at the input of FC_7.

So, f_4 frequency is obtained from the output of FC_7 and is applied as a strong pump beam to the FC_8. Light from input optical channel B acts as a weak probe beam to the FC_8. A_3 is biased by f_3 frequency and A_4 by f_1 frequency. Now, when B=f_1, then f_1 frequency is obtained from the output of FC_8 then A_3 passes this frequency but A_4 reflects it and is collected by a circulator C_4. It is then applied as a strong pump beam to the FC_{10}. A weak probe beam of f_2 frequency is present at the input of FC_{10}. So, f_2 frequency is obtained from the output of FC_{10} and it appears at the output optical channel O_2. When B=f_2, then f_2 frequency is obtained from the output of FC_8 and A_3 and A_4 both passes this frequency and is applied as a strong pump beam to the FC_9. A weak probe beam of f_1 frequency is present at the input of FC_9. As a result, f_1 frequency is obtained from the output of FC_9 and it finally appears at O_2. When B=f_3, then this frequency of light is obtained from the output of FC_8 and is reflected by A_3. Then, it is collected by the circulator C_3 and appears at O_2.

The truth table of quantum optical tristate CNOT gate with frequency encoded qubits is given in Tab.2. This table supports the logic of Tab.1. Here the output frequency at O_1 remains the same as that of the input frequency applied at A. The output frequency O_2 also remains the same with that of the input at B when the input frequency at A is $f_1(0)$. When $f_2(1)$ or $f_3(\overline{1})$ frequency of light signal is applied at the input A, the frequency applied at the input B is changed by following a tristate NOT operation and appears at the output optical channel O_2.

Tab.2 Truth table of tristate CNOT gate with frequency encoded qubits

Input frequencies		Output frequencies	
A	B	O_1	O_2
$f_1(0)$	$f_1(0)$	$f_1(0)$	$f_1(0)$
$f_1(0)$	$f_2(1)$	$f_1(0)$	$f_2(1)$
$f_1(0)$	$f_3(\overline{1})$	$f_1(0)$	$f_3(\overline{1})$
$f_2(0)$	$f_1(0)$	$f_2(1)$	$f_1(0)$
$f_2(0)$	$f_2(1)$	$f_2(1)$	$f_3(\overline{1})$
$f_2(0)$	$f_3(\overline{1})$	$f_2(1)$	$f_2(1)$
$f_3(\overline{1})$	$f_1(0)$	$f_3(\overline{1})$	$f_2(1)$
$f_3(\overline{1})$	$f_2(1)$	$f_3(\overline{1})$	$f_1(0)$
$f_3(\overline{1})$	$f_3(\overline{1})$	$f_3(\overline{1})$	$f_3(\overline{1})$

To give a discussion about the result of operation, it can be said that a proper truth table for quantum optical tristate CNOT gate is proposed and then using this truth table, the gate matrix for tristate CNOT gate is developed. The gate matrix is unitary and reversible in nature, which is very much essential for quantum operations. After that, quantum optical tristate CNOT gate is implemented using SOA based optical switches. In our proposed tristate system, '0' state is encoded by frequency f_1, '1' state by frequency f_2 and '$\overline{1}$' state by frequency f_3. The current work is a proposal of two qubit tristate CNOT gate. In the same way, two qubit quaternary state CNOT gate can also be proposed.

To conclude the description about the scheme, it can be said that the above implementation scheme of frequency encoded tristate CNOT gate, the XGM character of SOA has massively been used. The novelty of the system is the inclusion of tristate operation with light. As SOA is used as optical switching device, high speed of operation (beyond GHz) is expected very easily. Again, because of accommodation of tristate operation, the domain of parallel information processing is also extended far. The whole system is frequency encoded system and therefore the signal to noise ratio is quite high and the bit error rate is quite low. From the power consumption point of view, this is also advantageous as the SOAs require very small power for their operation.

Acknowledgements

The authors acknowledge the Department of Science and Technology (Govt. of India) for providing INSPIRE fellowship to one of the authors.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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