Investigation on the optical focusing effect of Fresnel biprism*

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Fresnel biprism has been applied to the design of plasmonic meta-lenses recently. In order to promote these applications and understand the physics behind them, in this paper we investigate the focusing effect of Fresnel biprism from the perspective of information optics and geometrical optics. The expression for optical field intensity describing the focusing effect of Fresnel biprism is derived according to the relationship between the impulse response function and the optical field. Then the formula of the focal length is achieved. Furthermore, the Fresnel biprism focusing experiment is performed. Specially, the optical field intensity distribution is measured and the corresponding intensity along the axis is obtained. The results show that the focusing effect depends on the base angle, refractive index and base length of the biprism. There exists axial resonance effect in the axial intensity. The experimental results are in accordance with the theoretical results. These results could be valuable to the applications of Fresnel biprism in designing large depth of focus plasmonic meta-lenses.

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Fresnel biprism[1] is an important optical element formed by two thin equal prisms joined at the base, which can be applied to measure the wavelength of light via interference under sodium lamp illumination with a single slit. Biprism interference experiment was performed by Fresnel in 1826[1,2]. Biprism interference experiment as a standard experiment has attracted much attention. Many studies have been devoted to the placement method of biprism[3], the measurement of virtual light distance[4] and the adjustment skill[5]. In 2013, DOBLAS et al[6] investigated the axial resonance of biprism interference under the extended incoherent illumination. Over the last decade, the biprism has been adopted to perform phase imaging[7], temporal interference[8], and digital holography microscopy[9,10]. In order to promote the application of phase imaging based on biprism and understand the physics behind it, ZHANG et al[11] discussed the biprism interference in the framework of information optics and presented the influence of single-slit width and the distance between the single-slit and the biprism on the quality of the interference pattern. Traditional phase contrast imaging scheme based on digital holographic microscopy requires two separate coherent beams, which leads to poor phase stability, and then the two beams are coherent superposition through Mach-Zehnder interferometer, in which the intensity ratio of the two beams should be adjusted precisely. While the digital holographic microscopy and the phase imaging based on biprism interference possess self-referencing and better stability properties due to the biprism itself can distinguish and superimpose the coherent beams[12]. JAFARFARD[13] presented a stable quantitative phase measurement technique where each wavelength was separated into three beams using Fresnel biprism. JOGLEKAR et al[14] investigated large field of view off-axis quantitative phase contrast microscopy by hologram multiplexing utilizing multiple Fresnel biprisms. Different from conventional optical lenses made of optical glasses, plasmonic metalenses are made of metasurface materials that are able to shape the amplitude and the phase of light with a high spatial resolution and subwavelength focusing in compact imaging system[15]. Recently, different from the biprism interference, the large depth of focus plasmonic meta-lenses has been designed based on the biprism without requirement of single slit[13]. Thus, it is necessary to reveal the physics behind the focusing effect of biprism so as to understand the operation principle of large depth of focus plasmonic metalenses based on biprism. In this paper, we intend to study the focusing effect and diffraction of biprism from the perspective of Fourier optics and present the formula of the focal length of biprism.

The schematic of experimental setup for performing the Fresnel biprism focusing effect is illustrated in Fig.1. The light beam generated by He-Ne laser passes through a beam expander and thus the collimated beam is

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achieved. Then the collimated light is incident vertically on a thin Fresnel biprism, and the charge coupled device (CCD) placed at the detection plane is used to capture the intensity pattern modulated by the biprism.

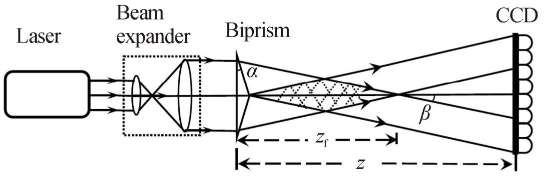

Fig.1 Schematic diagram of implementing the focusing effect of biprism

For the sake of simplicity, we assume the collimated light can be regarded as coherent plane light, whose optical field is denoted as E_0. Considering the geometrical structure, the transmission function of the thin biprism can be expressed as[8]

$$T(x_0)=\begin{cases}\exp[-\mathrm{i}k(n-1)\alpha x_0], & 0<x_0<a\\ \exp[\mathrm{i}k(n-1)\alpha x_0], & -a<x_0<0\end{cases}, \tag{1}$$

where x_0 indicates the transverse position of the thin biprism, n, α and $2a$ are the refractive index, base angle and base length of the thin biprism, respectively, and k=2π/λ is the wavenumber where λ is the wavelength of incident light.

Under the paraxial approximation, the impulse response function from the biprism to the detection plane can be given by[16,17]

$$h(x,x_0)=\sqrt{\frac{k}{\mathrm{i}2\pi z}}\exp\left[\mathrm{i}kz+\frac{\mathrm{i}k}{2z}(x-x_0)^2\right], \tag{2}$$

where z is the distance between the biprism and CCD, and x is the transverse coordinate of CCD detection plane.

In terms of Fresnel diffraction theory[14], the optical field modulated by the thin biprism on the detection plane can be written as

$$E(x)=\int E_0T(x_0)h(x,x_0)\mathrm{d}x_0. \tag{3}$$

Substituting Eqs.(1) and (2) into Eq.(3), the optical field on the detection plane can be expressed as

$$\begin{aligned}E(x)=&\sqrt{\frac{k}{\mathrm{i}2\pi z}}E_0\exp(\mathrm{i}kz)\times\\&\{\int_{-a}^{0}\exp[\mathrm{i}k(n-1)\alpha x_0+\frac{\mathrm{i}k}{2z}(x-x_0)^2]\mathrm{d}x_0+\\&\int_{0}^{a}\exp[-\mathrm{i}k(n-1)\alpha x_0+\frac{\mathrm{i}k}{2z}(x-x_0)^2]\mathrm{d}x_0\}.\end{aligned} \tag{4}$$

Then, in terms of Fresnel integral[14],

$$C(w)=\int_0^w\cos\left(\frac{\pi}{2}\tau^2\right)\mathrm{d}\tau,$$

$$S(w)=\int_0^w\sin\left(\frac{\pi}{2}\tau^2\right)\mathrm{d}\tau. \tag{5}$$

Eq.(4) can be further written as

$$\begin{aligned}E(x)=&\sqrt{\frac{k}{\mathrm{i}2\pi z}}E_0\exp\{-\frac{\mathrm{i}k}{2}[(n-1)^2\alpha^2z-\\&2(n-1)\alpha x-2z]\}\times\{[C(w_1)-C(w_2)]+\mathrm{i}[S(w_1)-S(w_2)]\}+\\&\sqrt{\frac{k}{\mathrm{i}2\pi z}}E_0\exp\{-\frac{\mathrm{i}k}{2}[(n-1)^2\alpha^2z+\\&2(n-1)\alpha x-2z]\}\times\{[C(w_3)-C(w_4)]+\\&\mathrm{i}[S(w_3)-S(w_4)]\},\end{aligned} \tag{6}$$

where

$$w_1=\sqrt{\frac{2}{\lambda z}}\left[x-(n-1)\alpha z+a\right],$$

$$w_2=\sqrt{\frac{2}{\lambda z}}\left[x-(n-1)\alpha z\right],$$

$$w_3=\sqrt{\frac{2}{\lambda z}}\left[x+(n-1)\alpha z\right],$$

$$w_4=\sqrt{\frac{2}{\lambda z}}\left[x+(n-1)\alpha z-a\right]. \tag{7}$$

Therefore, the intensity distribution of the optical field on the detection plane is given by

$$\begin{aligned}I(x)=\left|E(x)\right|^2=&|\sqrt{\frac{k}{\mathrm{i}2\pi z}}E_0\exp\{-\frac{\mathrm{i}k}{2}[(n-1)^2\alpha^2z-\\&2(n-1)\alpha x-2z]\}\times\{[C(w_1)-C(w_2)]+\mathrm{i}[S(w_1)-S(w_2)]\}+\\&\sqrt{\frac{k}{\mathrm{i}2\pi z}}E_0\exp\{-\frac{\mathrm{i}k}{2}[(n-1)^2\alpha^2z+\\&2(n-1)\alpha x-2z]\}\times\{[C(w_3)-C(w_4)]+\\&\mathrm{i}[S(w_3)-S(w_4)]\}|^2.\end{aligned} \tag{8}$$

According to Eqs.(4)—(8), we can perform the numerical analysis to exhibit the focusing effect of biprism. The numerical simulation results are shown in Figs.2 and 3. Fig.2 shows the normalized axial intensity of optical field, for different biprism base angles α, versus the distance z between the biprism and CCD. The parameters are chosen as x=0, λ=632.8 nm, n=1.52, and a=2 mm, and α=0.007 5, 0.015, 0.025 corresponding to Fig.2(a)—(c), respectively.

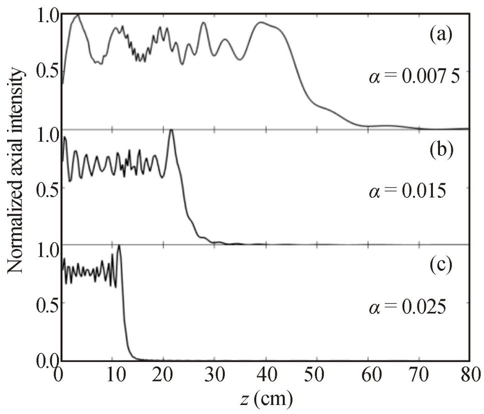

Fig.2 Normalized axial intensity as a function of the distance *z* between the biprism and CCD for different biprism base angles *α*

Analogy to the focusing of convex lens, we take the maximum axial distance, where the axial intensity tends to the minimum, as the focal length of biprism. The focal lengths of biprism corresponding to Fig.2(a)—(c) are about 51.4 cm, 25.7 cm, and 14.6 cm, respectively. From Fig.2, we can see that as the base angle of biprism increases, the focal length of biprism increases.

The normalized axial intensity of optical field, for different base lengths $2a$, as a function of the axial distance z is shown in Fig.3. The parameters for Fig.3 are selected as x=0, λ=632.8 nm, n=1.52, α=0.015, and a=0.5 mm, 1 mm, 2 mm corresponding to Fig.3(a)—(c), respectively. The focal lengths of biprism corresponding to Fig.3(a)—(c) are about 6.4 cm, 12.9 cm and 25.7 cm, respectively. From Fig.3, it can also be seen that as the wider the base length of biprism, the longer the focal length of biprism. From Figs.2 and 3, it can be found that there exists the axial resonance of intensity along the z axis.

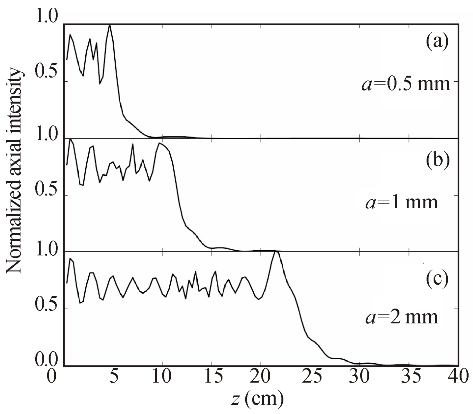

Fig.3 Normalized axial intensity as a function of the distance *z* between the biprism and CCD for different biprism base lengths 2*a*

The focusing effect can be explained as follows. In terms of Eqs.(7) and (8), when $z=a/[(n-1)/\alpha]$ is satisfied, the axial intensity tends to the minimum value which corresponds to the maximum axial distance that characterizes the superposition of beams superimposed by the biprism. Hence the focal length of biprism z_f can be characterized by this maximum axial distance. The formula of focal length can also be obtained in a geometric approach. With the aid of the geometry of biprism shown in Fig.1, we can obtain the following expressions according to Snell law

$$n\sin\alpha = \sin(\alpha+\beta), \tag{9}$$

$$a = z_f \tan\beta. \tag{10}$$

Under the paraxial approximation, the angle related to biprism satisfies

$$\sin\alpha \doteq \alpha,\ \ \sin(\alpha+\beta) \doteq \alpha+\beta,\ \ \tan\beta \doteq \beta. \tag{11}$$

Considering the above approximations as Eq.(11) and Eqs.(9) and (10), the formula for the focal length of biprism can also be obtained, which is written as

$$z_f = \frac{a}{(n-1)\alpha}. \tag{12}$$

In order to verify the above theoretical analysis, we perform the experiment to exhibit the focusing effect of biprism. We carry out the experiment in terms of the experimental scheme illustrated in Fig.1. The He-Ne laser (DH-HN1200, Daheng Optics) is used to generate coherent light of wavelength of 632.8 nm. The coherent beam passes through the beam expander (Omo 103, BOCI) to form the collimated light. Then the collimated light impinges on the biprism (WSZ-9, Tianjin Tuopu, the base angle of 0.007 6, refractive index of 1.46, and base length of 4 mm). In addition, a black and white CCD (Mintron, MTV-1881EX, 795×596 pixels with pixel size of 8.6 μm×8.3 μm, exposure time of 40 ms) is used to capture the light modulated by the biprism.

Fig.4 shows several typical intensity distributions of diffracted light fields registered by CCD. The axial distance z between CCD and biprism for Fig.4(a)—(h) is 10 cm, 15 cm, 20 cm, 25 cm, 30 cm, 35 cm, 45 cm, and 64 cm, respectively. To better illustrate the problem, the pseudo-color images corresponding to Fig.4 are shown in Fig.5.

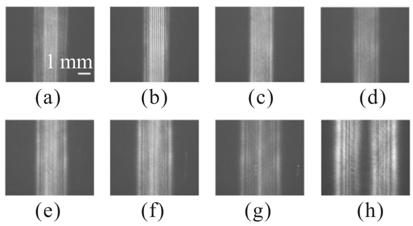

Fig.4 Typical intensity patterns of optical fields captured by CCD (The axial distance *z* between CCD and biprism corresponds to (a)—(h) is 10 cm, 15 cm, 20 cm, 25 cm, 30 cm, 35 cm, 45 cm, and 64 cm, respectively)

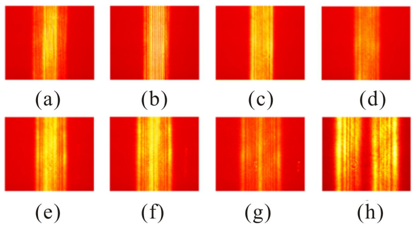

Fig.5 The pseudo-color images corresponding to Fig.4

From Figs.4 and 5, we can see that as the axial distance z increases, the intensity distribution modulated by biprism goes through the variations from partial coincidence, entirely coincidence to partial coincidence and separation again. It should be noted that the faint coherent noise (circular diffraction patterns) in Figs.4 and 5 may be caused by the diffraction of small defects or dust on the optical elements.

The measured axial intensity (x=0) of optical field versus the axial distance z is shown in Fig.6. It can be seen from Fig.6 that the resonance effect of intensity along the z axis exists, which is in accordance with the theoretical expectation. The measured focal length of biprism is 56.8 cm which is nearly consistent with the theoretical value calculated in terms of Eq.(12).

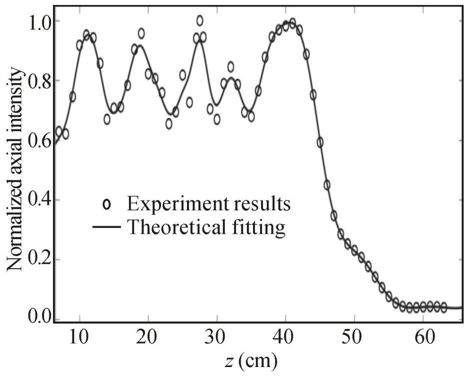

Fig.6 Normalized axial intensity as a function of the axial distance *z*

In summary, the focusing effect of Fresnel biprism from the perspective of information optics and geometrical optics is investigated. In terms of the relationship between the impulse response function and the optical field, the optical field intensity expression describing the focusing effect of Fresnel biprism is derived. Correspondingly, the formula of the focal length is achieved. Meanwhile, the formula is also analyzed from the geometric point of view. Furthermore, the Fresnel biprism focusing experiment is performed. It is shown that the focusing effect depends on the angle, the refractive index and base length of the biprism. The axial intensity exhibits the property of axial resonance. The experimental results are consistent with the theoretical expectation. These results can be useful in designing large depth of focus plasmonic meta-lenses based on biprism.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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