

# Research on the design of metalens with achromatic and amplitude modulation\*

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Metalenses are two-dimensional planar metamaterial lenses, which have the advantages of high efficiency and easy integration. However, most metalenses cannot modulate the light intensity, which limits their applications. To deal with it, taking advantage of flexible regulation of the beam amplitude and phase by the metalens, the geometric phase method is selected to design the dual-function metalens. It can effectively eliminate chromatic aberration in a visible light band from 535 nm to 600 nm and achieve amplitude modulation. After transmitting the metalens, the amplitudes of the beam respectively turn into 0.2 and 0.9. In this way, the amount of transmission of metalens in the preset band can be quantitatively controlled. According to the distribution characteristics of light diffraction intensity, the metalens designed can play a dual modulation role of achromatism and interference double-beam equilibrium in the paper, to meet the needs of miniaturization and integration of the optical system. The achromatic and amplitude-modulated metalens will have great application potential in optical holographic imaging and super-resolution focusing.

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The lens is a very important basic element in optical applications. Most conventional lenses are mainly made of glass and modulate the phase of the outgoing optical field by the cumulative effect of the optical range. Due to the limitations of the modulation principle and the characteristics of the materials, a bulky and complex optical system with multiple lens combinations is necessary to avoid a small field of view and large chromatic aberrations<sup>[1,2]</sup>. It is not conducive to miniaturization and integration of optical systems.

Metamaterials are artificial structural materials based on subwavelength structures, which were first proposed by SMITH<sup>[3]</sup> in 2001. It has flexible modulation of amplitude, phase, and polarization. It has also shown great potential for applications in negative refraction<sup>[4,5]</sup>, optical stealth<sup>[6,7]</sup>, and high-resolution imaging<sup>[8,9]</sup> in recent years.

In 2011, CAPASSO's team used their design of the V-shaped antenna metasurfaces to clarify the refraction/reflection law for metasurfaces, also known as the generalized refraction and reflection law<sup>[10]</sup>. Subsequently, the metasurfaces have caused a research upsurge among scholars. Metalens based on metasurfaces mechanisms has been developed then.

Initially, most metalenses operated at a single wavelength or multiple discrete wavelengths<sup>[11,12]</sup>. For example, CAPASSO's team designed a three-wavelength confocal metalens at 1 300 nm, 1 550 nm, and 1 800 nm,

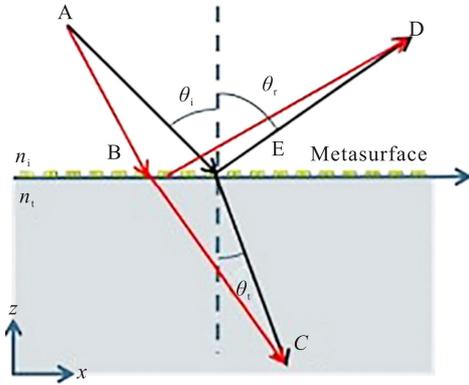
considering the dispersion of different wavelengths for phase compensation<sup>[11]</sup>. However, the achromatic range of conventional optical systems often applies to the entire visible and near-infrared light range. To achieve a broadband achromatic focusing metalens, dispersion compensation is required to focus light of different wavelengths. CAPASSO's team designed achromatic metalens based on titanium dioxide nanopillars with excellent achromatic focusing in visible light range from 490 nm to 550 nm<sup>[13]</sup>. In 2017, ZHU's team designed an achromatic metalens in the near-infrared band between 1 200 nm and 1 650 nm with an operating bandwidth of up to 450 nm by combining the integrated resonance and geometric phase<sup>[14]</sup>. In 2020, ZHAO et al<sup>[15]</sup> designed a broadband achromatic sub-diffraction focusing metalens by modulating the amplitude in the terahertz band. ARBABI et al<sup>[16]</sup> presented an achromatic metalens with arbitrary dispersion control by using cascaded metasurfaces (2D arrays of sub-micron scatterers) to direct light along predetermined trajectories in the same year. Most of the reported metalenses can be applied in the visible and near-infrared bands, which can achieve achromatic function well, but there are few metalenses with amplitude-modulation reported. In conventional optical systems, spatial light modulators are often used to regulate the beam amplitude, which inevitably leads to a more complex optical system structure. In this paper, based on the reported metalenses, the geometric phase method is

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used to modulate the beam information, which can achieve an amplitude modulation of 0.9 and 0.2 for the outgoing beam after the metalens and achieve a broadband achromatic range between 535 nm and 600 nm, with a constant focal length of 500  $\mu\text{m}$ . This paper provides a simple method for broadband achromatic amplitude-modulated metalens, which has application value in super-resolution focus imaging.

The derivation of the Fermat principle of anomalous refraction and reflection considers the extremum of the light range as the transmission path. It was first verified theoretically by CAPASSO et al in 2011 and was named as the generalized Snelle's law<sup>[10]</sup>. A metalens is placed between the interfaces of the two media from a single-dimensional perspective, giving the incident wave an abrupt phase shift to get a position-dependent abrupt phase shift  $\phi(x)$ , as shown in Fig.1.



**Fig.1 Generalized Snell's law**

By acquiring the additional phase with a certain gradient along the  $x$ -axis, the incident light wave from point A through points B and E arrived at the refraction point C and the reflection point D, respectively. The refraction/reflection paths have the same phase shift, and can be expressed as

$$\begin{cases} k_0 n_i \sin \theta_i dx + \varphi + d\varphi = k_0 n_i \sin \theta_r dx + \varphi \\ k_0 n_i \sin \theta_i dx + \varphi + d\varphi = k_0 n_t \sin \theta_t dx + \varphi \end{cases} \quad (1)$$

where  $k_0 = 2\pi/\lambda$  is the number of waves in a vacuum,  $\theta_i$  is the angle of incidence,  $\theta_r$  is the anomalous angle of reflection,  $\theta_t$  is the anomalous angle of refraction,  $n_i$  and  $n_t$  are the refractive indices of the two media respectively,  $dx$  is the distance between the two incident light beams entering the metasurfaces, and  $d\varphi$  is the phase difference between two incident light beams entering the metamaterial surface.

The refraction angle is no longer 0 even if the incident light enters the metasurfaces perpendicularly. The generalized Snelle's law can be derived from Eq.(1) as follows

$$\begin{cases} \sin \theta_r - \sin \theta_i = \frac{1}{k_0 n_i} \frac{d\varphi}{dx} \\ n_t \sin \theta_t - n_i \sin \theta_i = \frac{1}{k_0} \frac{d\varphi}{dx} \end{cases} \quad (2)$$

where  $d\varphi/dx$  is the gradient of the phase mutation.

It can be seen that the introduction of the phase gradient affects the refraction/reflection phenomenon of the beam, providing a new way to modulate light beams with metamaterials by using the superposition of basic units with different characteristics at different positions to change the wave vector of the refracted beams. This new method introduces a new degree of freedom, which effectively manipulates the phase, amplitude, and polarization of the desired outgoing light wave. In this paper, two mutually orthogonal cuboids are selected as the base units for the structural arrangement to achieve the required electromagnetic response of the achromatic amplitude-modulation metalens. The geometric phase (Pancharatnam-Berry phase) method is used to manipulate the shape and orientation angle of the structure of the unit to achieve effective modulation of phase and amplitude. To achieve a focusing performance consistent with the operation of a spherical lens, the base phase of the metalens needs to satisfy

$$\phi(x, y) = 2k\pi + \frac{2\pi}{\lambda} (f - \sqrt{x^2 + y^2 + f^2}), \quad (3)$$

where  $k$  is the number of waves,  $\lambda$  is the incident light wavelength of the metalens,  $\sqrt{x^2 + y^2}$  is the distance between the center of the metalens and any point on the structure, and  $f$  is the focal length of the metalens.

To arrange the overall metalens structure, we use Matlab software to obtain the basic phase distribution of the metalens in this paper. When the electromagnetic wave is incident positively along the  $z$ -axis, the polarization direction of the circularly polarized wave follows the direction of the two main axes of the base units. When the base units are rotated by an angle  $\theta$ , the outgoing light wave gets an attached phase ( $\pm i \cdot 2\theta$ ) and the chirality of the outgoing light wave is opposite to that of the incoming light wave. Thus, when left-handed circularly polarized light is incident, the conversion to right-handed circularly polarized light is achieved by rotating the base units at each  $(x, y)$  coordinate on the metalens, which can obtain the required compensated phase value for the achromatic amplitude-modulation metalens. Rotating around the center of the structure can provide a wider phase coverage, which can fully cover  $2\pi$  phase values. The rotation angle of each base unit on the metalens needs to satisfy

$$\theta(x, y) = \frac{\pi}{\lambda} (f - \sqrt{x^2 + y^2 + f^2}). \quad (4)$$

The amplitude  $E_0$  and phase  $k \cdot r - \omega t$  of  $E = E_0 e^{i(k \cdot r - \omega t)}$  in electromagnetic waves can be modulated by combining the fundamental phase in the geometric phase with the compensated phase. By optimizing the shape and spatial distribution of blocks and setting their heights, the phase and the amplitude of the metalens can be separately controlled. And the double-layer structure is chosen to increase the range of phase and amplitude. Firstly, we set the first target library with any one of the parameters of length and width in two mutually orthogonal rectangles

as the first target parameter, and kept all other parameters constant to achieve linear modulation of amplitude. And we did the same thing to the other three length and width parameters. Secondly, a second target library was set and two mutually orthogonal rectangles were used as the base units to rotate around the central axis to achieve the required target phase for linear regulation. The geometrical parameters of the base units and the angular values of the required rotation were established according to the complex amplitude distribution, which finally corresponded to the library.

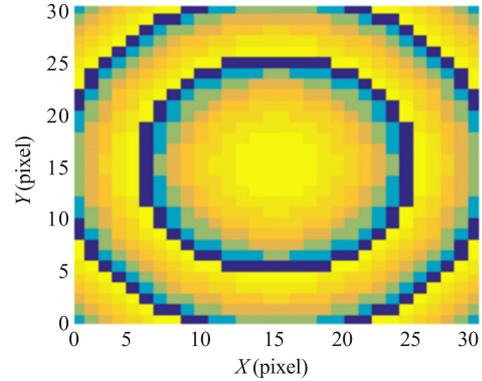
For the functional units of the metalens, we chose silicon material for the simulation. Depending on the geometrical phase characteristics, the geometrical parameters and the rotation angle of the base units provided information on the complex amplitude distribution. Setting the operating wavelength can adjust the length and width of the first and second dielectric blocks and optimize the height of the base units to let the electrode resonance peak and the dipole resonance peak of the functional units coincide. By setting any of the four parameters of the length and width and fixing the other parameters at the same time, the data of parameters and the target amplitude can be obtained.

Next, we rotated the functional units with the  $z$ -axis perpendicular to plane  $xoy$  and the linear relationship with the target phase by modulating the rotation angle with the  $x$ -axis. Finally, the required amplitude  $A_i$  was obtained and the unit's parameter  $W_i$  at this point was recorded, fixing the parameters at the same time. The phase  $\varphi_i$  was obtained and the rotation angle  $\theta_i$  at this point was recorded. The complex amplitude distribution was obtained according to the obtained parameter  $W_i$  and the rotation angle  $\theta_i$  as

$$E_i = A_i \times \exp(-j\varphi_i). \quad (5)$$

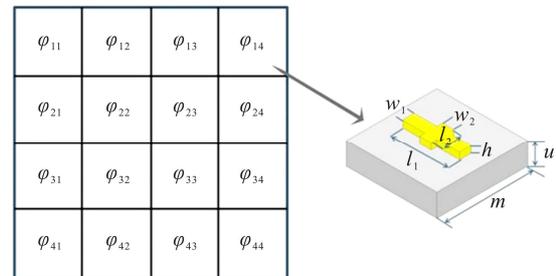
In this way, we obtained the corresponding amplitude and fundamental phase by separately selecting one of the unit geometry parameters for sweeping the parameters and fixing the others. After recording, we performed a revolution sweep of the unit column with the geometric parameters corresponding to this phase. Therefore, the change of phase is caused by the base phase  $\varphi_1$  aroused by geometric parameters changing and the modulation phase  $\varphi_2$  obtained by rotating the functional unit structure and the angle  $\theta$  of the  $x$ -axis. The changes of  $\varphi_1$  and the unit geometry parameters can be obtained directly from the CST software simulation. The relationship between  $\varphi_2$  and the units rotation angle is  $\varphi_2=2\theta_i$  ( $\theta \in (0, \pi)$ ). In the end, we can get the total phase  $\varphi=\varphi_1+\varphi_2$ . Through this decomposition of the modulated complex amplitude and then modulation, we can compensate for the material regret of the current inability to simultaneously record amplitude and phase. It can also suppress the noise and conjugate images. This direct modulation of the complex amplitude allows the acquisition of high diffraction efficiency three-dimensional displays. And the reduced pixel size of the metalens allows for a larger field of view. Fig.2 shows the distribution of the complex amplitude

under the metasurfaces designed in this paper. The axes represent pixels, and each pixel represents the nano-unit in the model. The different colors show the different distributions of complex amplitudes of different units.



**Fig.2 Distribution of complex amplitude**

To improve the large volume of the traditional achromatic optical system, the functional units selected for the design of the achromatic metalens are shown in Fig.3. We select dielectric silicon with a refractive index of 3.5 as two mutually orthogonal rectangular column materials and sweep the geometric parameters of the functional units for optimization when the left-handed circularly polarized light is incident. When  $l_1=350$  nm,  $l_2=150$  nm,  $w_1=60$  nm,  $w_2=60$  nm and height of units  $h=65$  nm, the units electrode and magnetic polariton resonance peaks coincide. The polarization efficiency can reach 88%.

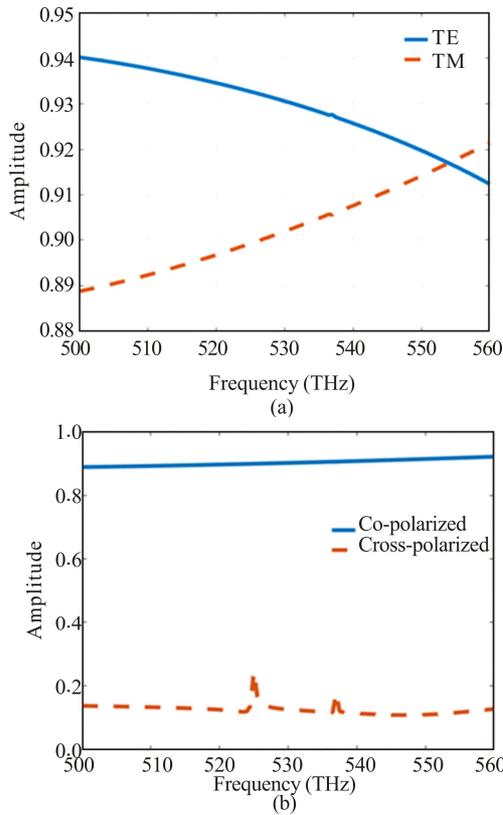


**Fig.3 Schematic diagram of phase distribution and oblique view of the basic units**

By simulating the functional units, we obtain the change in amplitude of the transmitted wave after the incidence of transverse electric (TE) and transverse magnetic (TM) waves. Both TE and TM waves can reach an amplitude of 0.91 when the incident wavelength is 540 nm as shown in Fig.4(a). In Fig.4(b), when the metalens base units are incident on a left-handed circularly polarized wave, the maximum of right-handed circularly polarized outgoing light wave amplitude exceeds 0.9 and the maximum of left-handed circularly polarized light wave amplitude exceeds 0.2, achieving effective amplitude modulation.

Due to the high refractive index contrast between silicon and air, the incident light is mainly confined in the silicon with a high refractive index. Meanwhile, silicon

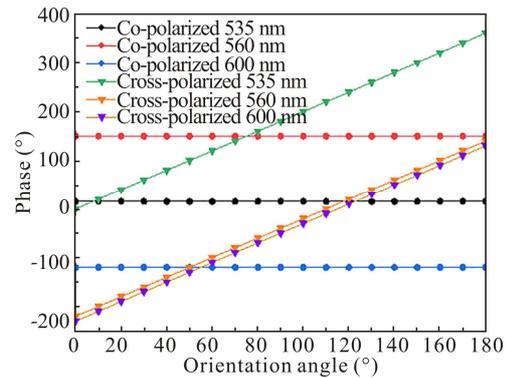
has a strong absorption effect on the visible light band. According to the waveguide mode resonance effect of the metalens, it can produce high transmittance to the incident light at different wavelengths, so it meets the requirements of transmission metalens. By selecting the functional units with the material properties, the results obtained through simulation meet the amplitude modulation requirements and satisfy the overall structural arrangement of the transmissive amplitude-modulation metalens. The complex permittivity of silicon is taken from Palik's data and its transmissivity is influenced by the geometrical parameters of the functional units.



**Fig.4 (a) Amplitude curves of TE and TM waves; (b) Right/left-handed polarized amplitudes**

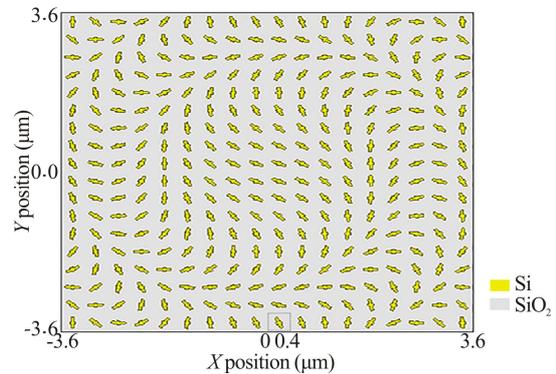
Based on the complex amplitude method of Huygens metasurfaces, the target phase and amplitude required by the function of the metalens element are achieved by modulating the geometrical parameters of the functional units and the overall rotation angle. We control amplitude through changes in the geometric parameters of the functional units. And in the vertical plane  $xoy$ ,  $z$ -axis and  $x$ -axis rotation angle from  $0$  to  $\pi$  can realize the control of phase, thus achieving arbitrary complex amplitude control. We also simulate the relationship between the transmitted phase and the rotation angle for two different polarization states at the incident wavelengths of 530 nm, 565 nm, and 600 nm respectively. As shown in Fig.5, the dots in the diagram are co-polarized states and the triangles are cross-polarized states. The phase of transmission

at different wavelengths in the same polarization state does not change with the angle of orientation. Passing through a metalens is like adding a constant phase, consistent with the effect of passing through a flat plate with uniform refractive index. However, the transmission phase and rotation angle of the cross-polarized state vary linearly, producing a focusing effect on the beam under the diffractive lens focusing principle.



**Fig.5 Phase responses of co-polarized/cross-polarized waves**

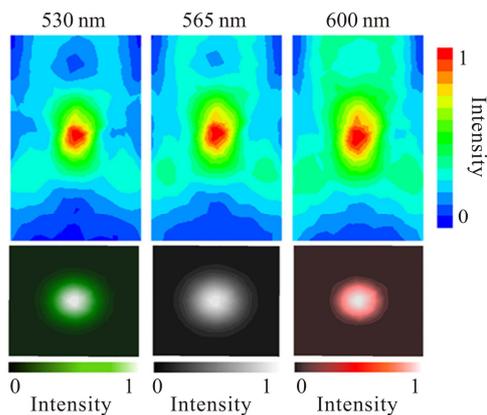
The required units' library is created by simulating the unit columns, and the layout of the metalens is designed according to a certain layout rule. The overall structure is laid out according to the determined amplitudes and phases in the corresponding units' library, as shown in Fig.6. Each unit is a  $0.4 \mu\text{m} \times 0.4 \mu\text{m}$  square, which avoids the crosstalk between the adjacent units and ensures the moderate function. For the Huygens principle, metalens is designed to have a thinner unit than conventional metasurfaces, thus facilitating the bulk processing of metalens for large-area applications, such as holographic displays and light field shaping.



**Fig.6 Top view of the overall structure of metalens**

To verify the performance of the designed metalens, the structure was simulated and verified utilizing the software CST. Due to the limitation of the computer, the metalens was set to have an aperture of  $15 \mu\text{m}$  and a focal length of  $30 \mu\text{m}$ . We selected the 535 nm, 550 nm, and 600 nm incident light wavelengths respectively in

the 535 nm to 600 nm band for discussion. Their focal lengths were calculated to be 28.7  $\mu\text{m}$ , 29  $\mu\text{m}$ , and 30.2  $\mu\text{m}$ , deviating from the theoretical calculation by only 3.62%, 2.12%, and 2.38% respectively. Low error proves that achromatic function is achieved. When the values of the lens aperture are certain, we compare the focal depths of two different incident light beams. It can be found that as the wavelength increases, the size of the focal spot in the  $z$ -axis direction is also increasing, which means the focal depth increases with the wavelength. According to the Rayleigh criteria, the focal depth is  $\Delta z = \lambda / (2NA^2)$ . Moreover, the numerical aperture ( $NA$ ) becomes larger as the focal length decreases, which proves that the focal point is not a 'point' but an 'ellipsoid'. Meanwhile, as shown in Fig.7, the metalens after amplitude modulation is well focused and has a concentrated focal spot compared to normal metalens without an amplitude modulation effect. The 0—1 in Fig.7 represents the intensity, and different colors represent different intensities.



**Fig.7 Focus intensity distributions in the  $y$ - $z$  plane and  $x$ - $y$  plane**

By simulating the units, we found that the TE and TM electromagnetic responses in the cross-polarized state in the outgoing light can both reach 0.9. When left-handed polarized light is incident on the functional units, the highest amplitude of the right-handed polarized outgoing light wave exceeds 0.9 and the highest amplitude of the left-handed polarized light wave exceeds 0.2, effectively achieving amplitude control. To increase the phase coverage, the lens was further optimized in this design to have both sides above and below the substrate. And both sides of the unit column have the same geometry so that the phase covers the  $2\pi$  phase value. This metalens not only improves transmission but also simplifies the complex structure of conventional optical systems, contributing to the future application of metalens in integrated optoelectronic systems and light field shaping, holographic displays, and other fields.

In summary, based on the geometric phase theory, this paper designed an amplitude-modulated metalens in visible light range utilizing silicon material, successfully

eliminating chromatic aberration and providing amplitude modulation between the incident wavelengths of 535 nm and 600 nm. The transmission values of the metalens are controlled to 0.2 and 0.9 respectively for the set waveband. This metalens with a double-modulation function can achieve the effect of eliminating chromatic aberration and interference double beam equalization, achieving the goal of one mirror for multiple uses, and promoting the integration of optical devices. It can also effectively improve the characteristics of traditional optical systems with complex optical paths and large sizes, meeting the development needs of miniaturization of optical devices and paving the way for the construction of integrated optical systems. This dual-function metalens with achromatic and amplitude modulation has wide values in both microscopic imaging and optical instrumentation.

## Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

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