Some study on dark and bright optical solitons in a real system with periodically distributed dispersion and nonlinearity

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We study the dynamics of short optical pulses in a real system with periodically distributed dispersion and the nonlinearity is governed by the higher order nonlinear Schrödinger equation (HNLSE) with linear and nonlinear gain (loss). Under specific parametric circumstances, where the dark and bright solitary waves are combined, a set of entirely new types of solitary waves with nonlinear chirp have emerged. For a properly intense optical pulse in the combined solitary waves, the binding of the bright and dark solitary waves is very strong. It is seen that by numerical simulation, these pooled types of solitary-like solutions show a high degree of stability while propagating over an extremely long distance in the considering system, even in the presence of a high degree of perturbation of the amplitude and white noise. All constraint relations on the physical parameters are explicitly shown to be related to the development and to the dynamical study of the chirped solitary like solution in the considering medium.
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The optical pulse or beam transmits through an optical fibre without altering its shape and size when the refractive nonlinearity of the material of the fibre medium, induced by the intensity distribution of the optical pulse, exactly compensates for the pulse dispersion, called soliton. Most of the time, the weak nonlinearity appearing from the Kerr effect is responsible for the development of soliton in the fibre of silica glass. It induces a change in refractive index directly proportional to the intensity of the optical pulse. In this case, the cubic nonlinear Schrödinger equation (NLSE) governs the transmission of solitary waves in an optical fibre[1]. For an ample explanation of the transmission of very short optical pulses in nonlinear media, the cubic NLSE must be rectified to comprise some higher order effects[2]. It is recognized that temporal optical solitary waves have recently been the heart of intensive explore due to their potential applications in ultra-long distance fibre optic communication systems and very fast switching devices. Mind has concentrated in the past few years on the analysis of NLSE, with the terms of group velocity dispersion (GVD), third order dispersion (TOD), fourth order dispersion (FOD), intermodal dispersion, self-steepening effect (SS), and stimulated Raman scattering (SRS)[3,4]. Recently, the higher order nonlinear Schrödinger equation (HNLSE) has been analysed in numerous ways (e.g., inverse scattering transform, Ablowitz-Kaup-Newell-Segur (AKNS) method, Hirota direct method, Darboux-Baclund transform, Painleve analysis, and conservation laws), and some types of exact solitary solutions have been found. During the past years, a number of papers have been published for a number of purposes, such as the transmission of solitons and interaction between the solitons[5], chirp kink similaritons with varying Raman effects[6], generation of chirped femtosecond solitary waves and double-kink solitary waves[7], and for the development of white noise functional solitary wave solutions[8], considering the nonlinear fibres with inhomogeneous nonlinearity and dispersion. Refs.[9,10] introduced the collective variable method to describe the transmission of a solitary wave in a vastly dispersed inhomogeneous fibre medium, and they demonstrated a transfixing periodicity in the soliton's chirp, amplitude, frequency, width and phase. Ref.[11] explored chirped bright and dark solitary type waves in a nonlinear electrical transmission line (NLET). In Refs.[12,13], the authors freshly solved NLSE with the SS effect to come across solitary-like solutions with nonlinear chirping. In Refs.[14,15], the authors studied the stability of the nonlinearly chirped solitons. More recently, the authors of Refs.[16,17] solved HNLSE with non-Kerr nonlinearities to find families of chirped solitary like solutions.

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Furthermore, the authors of Refs.[18,19] solved the generalised NLSE for the existence of chirped solitary like solutions in polynomial nonlinearity and non-Kerr law media. In this work, we presume an HNLSE with space-dependent coefficients, as well as the TOD, SRS, SS, and linear and nonlinear gain (loss), and come across the exact chirped solitary solutions of a completely innovative type. By employing numerical simulation, the dynamics of the chirped solitary waves are studied and the elevated solidity of the solitary-like solutions in the presence of crystal-clear perturbation terms was also investigated.

The mathematical model that describes the transmission of an optical solitary wave of undersized pulse width in an inhomogeneous single-mode optical fibre in the existence of both linear and nonlinear gain (loss) can be expressed as follows

$$\mathrm{i}\frac{\partial q}{\partial z}=\frac{\beta_2(z)}{2}\frac{\partial^2 q}{\partial t^2}+\mathrm{i}\frac{\beta_3(z)}{6}\frac{\partial^3 q}{\partial t^3}+R(z)q\frac{\partial|q|^2}{\partial t}+ S(z)\frac{\partial}{\partial t}\left(|q|^2 q\right)-\gamma(z)|q|^2 q+\mathrm{i}\Gamma(z)q+\mathrm{i}\Pi(z)|q|^2 q, \quad (1)$$

where $q(z, t)$ is the amplitude of the solitary wave, the self-determining coordinates z and t respectively symbolize the distance along the fibre and time, $\beta_2(z)$, $\beta_3(z)$, $R(z)$, $S(z)$, and $\gamma(z)$ stand for the space dependent coefficients of GVD, TOD, SRS, SS and the cubic nonlinear effect, respectively, and $\Gamma(z)$ and $\Pi(z)$ respectively symbolize the space dependent coefficients of linear and nonlinear absorption or amplification.

To resolve the chirped solitary like solution of Eq.(1), we presume the solitary like solution as follows

$$q(z,t)=A(z)\operatorname{sech}\left[\eta(z)\left(t-T(z)\right)\right]\exp\left[\mathrm{i}\Phi(z,t)\right], \quad (2)$$

$$\Phi(z,t)=\lambda(z)\ln\operatorname{sech}\left[\eta(z)\left(t-T(z)\right)\right]+ a(z)+b(z)t+c(z)t^2, \quad (3)$$

where $\lambda(z)$ represents the nonlinear chirp, and $A(z)$, $\eta(z)$, $T(z)$ and $\phi(z)$ respectively represent the amplitude, inverse width of the pulse, time position, and phase of the optical pulse. $a(z)$ and $b(z)$ respectively represent the initial phase and frequency of the pulse, and $c(z)$ represents the linear chirp effect.

Substituting Eq.(2) and Eq.(3) into Eq.(1), and then sorting out the real and imaginary parts after removing the exponential parts, we reach the expressions as follows

$$\lambda(z)=\lambda_0=const, \quad (4)$$

$$\frac{\mathrm{d}c}{\mathrm{d}z}=2\beta_2(z)c(z)^2-2\beta_3(z)b(z)c(z)^2, \quad (5)$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}z}=2\beta_2(z)c(z)\eta(z)-2\beta_3(z)b(z)c(z)\eta(z), \quad (6)$$

$$\frac{\mathrm{d}b}{\mathrm{d}z}=2\beta_2(z)c(z)b(z)+\beta_3(z)c(z)\eta(z)^2- \beta_3(z)c(z)\eta(z)^2\lambda_0^2-\beta_3(z)b(z)^2c(z), \quad (7)$$

$$\frac{\mathrm{d}a}{\mathrm{d}z}=\frac{1}{2}\beta_2(z)b(z)^2+\frac{1}{2}\beta_2(z)\eta(z)^2\left(\lambda_0^2-1\right)- \frac{1}{2}\beta_3(z)\eta(z)^2\left(\lambda_0^2-1\right)b-\frac{1}{6}\beta_3(z)b(z)^3, \quad (8)$$

$$\frac{\mathrm{d}T}{\mathrm{d}z}=-2\beta_2(z)c(z)T(z)+2b(z)c(z)\beta_3(z)T(z)- \beta_2(z)b(z)+\frac{1}{2}\beta_3(z)b(z)^2-\frac{1}{2}\beta_3(z)\eta(z)^2- \frac{\beta_3(z)c(z)}{\lambda_0}+\frac{1}{6}\beta_3(z)\eta(z)^2\lambda_0^2, \quad (9)$$

$$A(z)=\sqrt{\frac{\left(\lambda_0^2-2\right)\left(\beta_2(z)-b\beta_3(z)\right)}{2\gamma(z)}}\eta(z), \quad (10)$$

$$\Gamma(z)=\frac{1}{A(z)}\frac{\mathrm{d}A(z)}{\mathrm{d}z}-\beta_2(z)\eta(z)^2\lambda_0-\beta_2(z)c(z)+ \beta_3(z)b(z)\eta(z)^2\lambda_0+\beta_3(z)b(z)c(z), \quad (11)$$

$$\Pi(z)=\frac{3\lambda_0}{\lambda_0^2-2}\gamma(z)-S(z)b, \quad (12)$$

$$\left(3S(z)+2R(z)\right)A^2=\frac{\beta_3}{6}\left[\lambda_0^2-11\right]\eta^2\lambda_0, \quad (13)$$

$$2S(z)A^2+3\beta_3\eta^2\lambda_0=0. \quad (14)$$

Eq.(4) implies that the nonlinear chirp parameter of the solitary wave is stable while propagating all through the inhomogeneous fibre. From Eq.(9), we see that the rate of change of time position with the distance of the pulse depends on GVD, TOD, and also depends on the linear and nonlinear chirp effects and frequency of the pulse. The Eq.(10) implies that the amplitude of the pulse depends on the GVD, TOD, nonlinearity of the medium, inverse pulse width, and the nonlinear chirp effect of the pulse. From the above expressions, we can certainly uncover that once the linear chirp effect is ignored, i.e., $c(z)$=0, we have $b(z)=b_0=const$, $\eta(z)=\eta_0=const$. Thus, in the deficiency of a linear chirping effect, the optical pulse propagates down the fibre with an unvarying frequency and an unvarying inverse pulse width. If the pulse energy is defined as $E=\frac{A(z)}{\eta(z)}$, then from Eq.(10), we can terminate that the assuming system is not conservative. For picoseconds optical pulses, we can assume $S(z)$=0, and if we regard as the nonlinear chirp parameter λ_0=0, then from Eq.(12) we acquire $\Pi(z)$=0. Thus, the nonlinear gain is absolutely accountable for the nonlinear chirp effect, and so in the case of a single mode inhomogeneous optical fibre, we can organize the nonlinear gain by choosing the nonlinear chirp in the appropriate way. But it is vital to use even more diminutive optical pulses in fibre optic communication to boost the channel managing capacity and for high speed communication, and then we cannot overlook the SS term and consequently the nonlinear gain.

For nonlinear chirp parameter λ_0=0 from Eq.(12), we find $\Pi(z)$=$S(z)b(z)$ and thus the nonlinear gain depends on the space dependent coefficients of SS and the frequency of the ultra short optical pulse. According to Eq.(10) and Eq.(14), we can manipulate the nonlinear gain even for ultra short optical pulses by selecting the appropriate value of the nonlinear chirp effect in the proper parametric conditions. We can also bring to light that the change of nonlinear chirp disturbs the initial phase, time position, amplitude of the pulse and linear gain (loss) of the system directly. When $A(z)$>0 and $\eta(z)$>0, the solution in Eq.(2) is justifiable and according to this, we will acquire the conditions $\lambda_0^2 < 2$ for the bright solitary wave and $\lambda_0^2 > 2$ for the dark solitary wave for $(\beta_2 - b\beta_3)\gamma > 0$, and we will come across the conditions $\lambda_0^2 > 2$ for bright solitary wave and $\lambda_0^2 < 2$ for dark solitary wave for $(\beta_2 - b\beta_3)\gamma < 0$. Thus, the solution in Eq.(2) is authentic for both bright and dark solitary waves. From this aspect, it may unite both bright and dark solitary waves concurrently under the some circumstances, and they are transmitted concurrently all the way through an optical fibre in coalesced form. These solitary waves of entirely new born types might be referred to as pooled solitary waves. These types of solitary waves may be dissociated into bright and dark solitary waves.

The phase of the pulse of the solitary solution is given by

$$\Phi(z,t) = \lambda_0 \ln \operatorname{sech}\left[\eta(z)(t - T(z))\right] + a(z) + b(z)t + c(z)t^2. \tag{15}$$

And the allied chirping effect can be expressed as

$$\mathrm{d}\omega(z,t) = -\frac{\partial \Phi(z,t)}{\partial t} = \lambda_0 \eta(z) \tanh\left[\eta(z)(t - T(z))\right] - b(z) - 2c(z)t. \tag{16}$$

In this work, we assume a periodic distributed system with varying coefficients of GVD and TOD as follows

$$\beta_2(z) = \beta_{20}\cos(\sigma z), \tag{17}$$

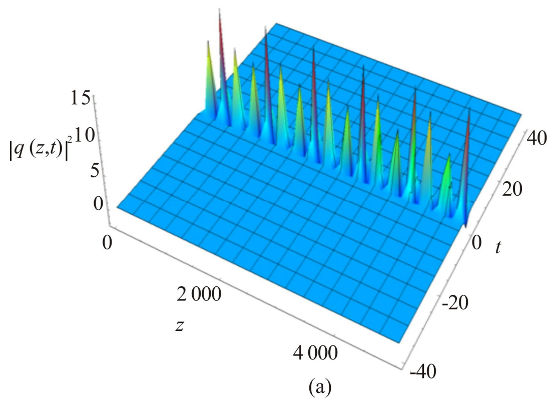
$$\beta_3(z) = \beta_{30}\cos(\sigma z), \tag{18}$$

and the coefficient of nonlinear term is given by

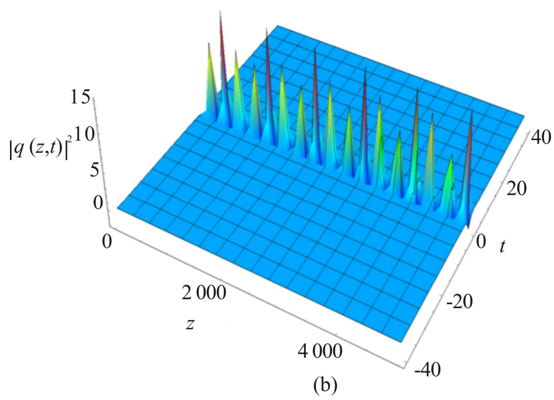
$$\gamma(z) = \gamma_0 \exp\left[\cos(\sigma z)\right], \tag{19}$$

where β_{20} and β_{30} are the constants, and the parameters γ_0 and σ describe the Kerr nonlinearity. The instantaneous occurrence of the space dependent coefficients of dispersions of the form in Eq.(17) and Eq.(18) and the space dependent coefficient of the nonlinear term of the form in Eq.(19) especially plays a crucial role in the development of such a pooled type of solitary wave. But for the stability of the propagation of such types of solitary waves, the binding of bright and dark solitary waves is very important and requires a proper intense optical pulse. For the analysis of the evolution characteristics of the chirped bright solitary like soliton, we offer the transmission plots in Fig.1 and Fig.2.

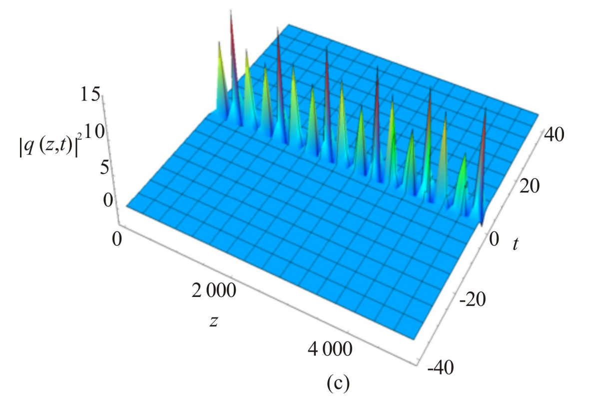
In the numerical plot, the chirped bright solitary-like solution with λ_0=0.5 for the case $(\beta_2 - b\beta_3)\gamma > 0$ in Fig.1(a) is shown. From the figure, we find that the evolution and stability of the pulse do not depend on the initial nonlinear chirp for sufficient pulse intensity. Due to the occurrence of the space dependent coefficients of GVD and TOD of the cosine type and the space dependent coefficient of nonlinear effect SPM of the exponential cosine type, a strange type of chirped solitary like solution is developed where dark and bright solitons are in allied and the advancement of such a type of solitary wave is very stable against the nonlinear chirp and also under a high degree perturbation of the amplitude and noise. For further exploration of the stability of the soliton-like solution, we have observed two types of numerical evolution under initial perturbation. In the first, we perturbed the amplitude to 90%, and finally we introduced 20% white noise in the second. The outcomes are depicted in Fig.1(b) and Fig.1(c), respectively.



(a)



(b)



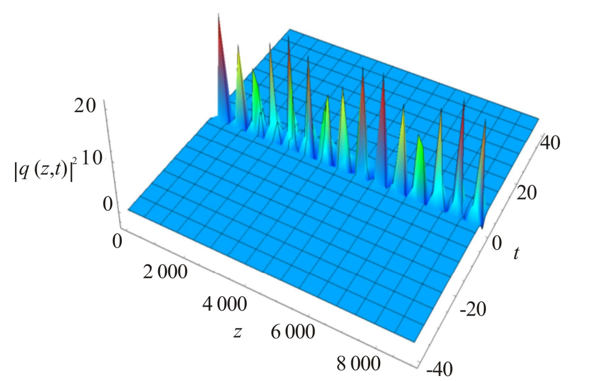
(c)

Fig.1 Progression of chirped bright solitary-like solution when (β_2−$b\beta_3$)γ>0, β_{20}=0.1, β_{30}=−0.1, γ_0=0.03, σ=2.09, $c(z)$=0 with (a) λ_0=0.5, (b) λ_0=0.5 and the amplitude is 90% of the exact solution, and (c) λ_0=0.5 and 20% white noise is added to the perturb solitary solution with 90% of the amplitude

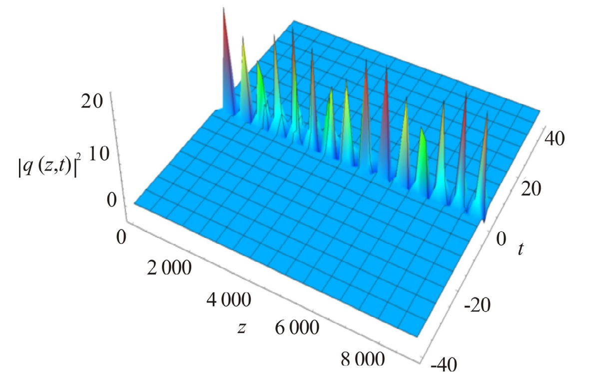
We revealed that small amplitude changes and the adding up of white noise have no effect on the stability of the solitary-like solution. Fig.1(c) illustrates the advancement of a chirped bright soliton-like solution when $(\beta_2 - b\beta_3)\gamma > 0$ with λ_0=0.5 and 20% white noise is added to the solitary solution, which is 90% perturb in amplitude of exact solitary solution. When $(\beta_2 - b\beta_3)\ \gamma < 0$, we provide numerical evolution with λ_0=2.1 in Fig.2(a). At first, we perturbed the amplitude to 90%, and finally we introduced 30% white noise in the second. The outcome is depicted in Fig.2(b). We notice that pulse progression is even now stable.

We provide the chirped dark solitary-like solution propagation plot when $(\beta_2 - b\beta_3)\gamma > 0$ with λ_0=1.5 in Fig.3, and it clearly shows the simultaneous existence of both dark and bright solitory waves. The choice of the proper intense optical pulse is also very important for dark solitary wave.

We provide the chirped dark solitary-like solution propagation plot when $(\beta_2 - b\beta_3)\gamma < 0$ with λ_0=1.2 as shown in Fig.4.



(a)



(b)

Fig.2 Progression of chirped bright solitary-like solution when (β_2−$b\beta_3$)γ<0, β_{20}=0.1, β_{30}=−0.1, γ_0=−0.03, σ=3, $c(z)$=0 with (a) λ_0=2.1 and (b) λ_0=2.1 and 30% white noise is added to the perturb solitary solution with 90% of the amplitude

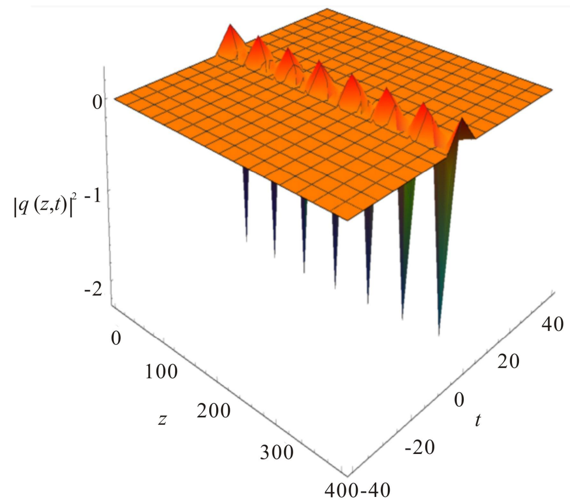


Fig.3 Progression of chirped dark solitary-like solution when (β_2−$b\beta_3$)γ>0, β_{20}=0.1, β_{30}=−0.1, γ_0=0.03, σ=2.09, $c(z)$=0, and λ_0=1.5

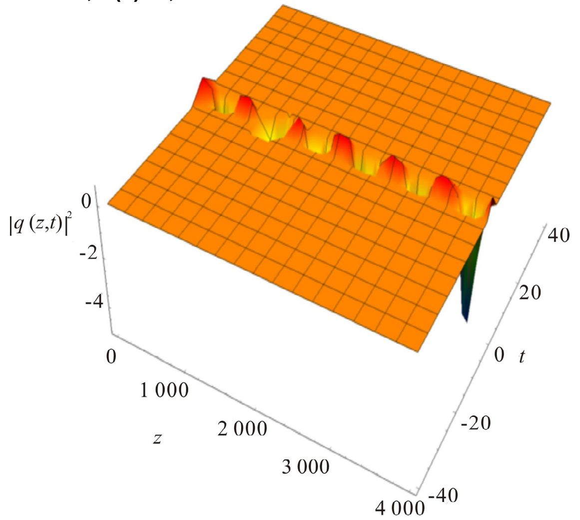


Fig.4 Progression of chirped dark solitary-like solution when (β_2−$b\beta_3$)γ<0, β_{20}=0.1, β_{30}=−0.1, γ_0=−0.03, σ=3, $c(z)$=0, and λ_0=1.2

The intensity profile of the combined solitary waves at z=0 is shown in Fig.5 with λ_0=0.5 and λ_0=1.5 when $(\beta_2 - b\beta_3)\gamma > 0$.

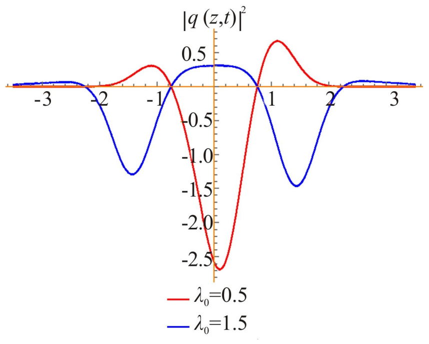


Fig.5 Intensity variation of chirped bright (red) and dark (blue) solitary-like solutions at *z*=0 when (β_2−$b\beta_3$)γ>0, β_{20}=0.1, β_{30}=−0.1, γ_0=0.03, σ=2.09, *c*(*z*)=0

In Fig.5, the curve of red color for λ_0=0.5 represents the bright solitary wave and the plot of blue color for λ_0=1.5 represents the dark solitary wave.

From Fig.5, we clearly notice that both bright and dark solitary waves are present and united. It is mentioned that the subsistence of the chirped solitary solution will be customised by the constriction relation in Eq.(11).

We have established an innovative type of chirped bright and dark solitary wave solutions by introducing nonlinear gain (loss) to the HNLSE together with periodic altering coefficients, where they are in pooled form. For picoseconds optical pulses, we can precisely manage the nonlinear gain (loss) by precisely selecting the initial nonlinear chirp of the true communication system. With appropriate parametric conditions, even for ultra short optical pulses, we can also manipulate the nonlinear gain by choosing the proper form of the nonlinear chirp parameter. The parametric conditions for the subsistence of such a type of solitary wave are also revealed. By way of simulation, we have uncovered that such a chirped solitary-like solution can propagate with high stability over a very long distance in the real fibre medium in the occurrence of a predetermined perturbation. This analysis may also offer insight into how we can supervise the nonlinear gain or loss in the transmission of ultra-short optical pulses by introducing an initial nonlinear chirp in an authentic system.

Statements and Declarations

The authors declare that there are no conflicts of interest related to this article.

References

[1] MERABI A, TRIKI H, AZZOUZI F, et al. Propagation properties of chirped optical similaritons with dual-power law nonlinearity[J]. Chaos, solitons & fractals, 2020, 140：110158.

[2] TRIKI H, ZHOU Q, BISWAS A, et al. Self-frequency shift effect for chirped self-similar solitons in a tapered graded-indexed waveguide[J]. Optics communications, 2020, 468：125800.

[3] HOSSEIN K, SADRI K, MIRZAZADEH M, et al. A high-order nonlinear Schrodinger equation with the weak non-local nonlinearity and its optical solitons[J]. Results in physics, 2021, 23：104035.

[4] PATHANIA S, GOYAL A, RAJU T S, et al. Chirped nonlinear resonant states in femtosecond fiber optics[J]. Optik, 2021, 227：166094.

[5] WU G Z, DIA C Q, WANG Y Y, et al. Propagation and interaction between special fractional soliton molecule in the inhomogeneous medium[J]. Journal of advance research, 2022, 36：63-71.

[6] TRIKI H, BENLALLI A, ZHOU Q, et al. Formation of chirped kink similariton in non-Kerr media with varying Raman effect[J]. Results in physics, 2021, 26：104381.

[7] KENGNE E, LAKHSSASSI A. Compensation process and generation of chirped femtosecond solitons and double-kink solitons in Bose-Einstein condensates with time-dependent atomic scattering length in a time-varying complex potential[J]. Nonlinear dynamics, 2021, 104：4221-4240.

[8] WANG B H, WANG Y. YF ractional white noise functional soliton solutions of a wick-type stochastic fractional NLSE[J]. Applied mathematics letters, 2020, 110：106583.

[9] RAZA N, HASSAN Z, SEADAWY A. Computational soliton solutions for the variable coefficient nonlinear Schrödinger equation by collective variable method[J]. Optical and quantum electronics, 2021, 53：400.

[10] ZHENG H, WU C, WANG Z, et al. Propagation characteristics of chirped soliton in periodic distributed amplification systems with variable coefficients[J]. Optik, 2012, 123：818-822.

[11] HOUWE A, ABBAGARI S, INC M, et al. Chirped solitons in discrete electrical transmission line[J]. Results in physics, 2020, 18：103188.

[12] TRIKI H, ZHOU K, BISWAS A, et al. Chirped optical solitons having polynomial law of nonlinear refractive index with self-steepening effect and nonlinear dispersion[J]. Physics letters A, 2021, 412：127698.

[13] TRIKI H, ZHOU Q, MOSHOKOA S P, et al. Chirped W-shaped optical solitons of Chen-Lee-Liu equation[J]. Optik, 2018, 155：208-212.

[14] DAOUI A K, AZZOUZI F, TRIKI H, et al. Propagation of chirped gray optical dips in nonlinear materials[J]. Optics communications, 2019, 430：461-466.

[15] HOUWE A, ABBAGARI S, ALMOHSEN B, et al. Chirped solitary waves of the perturbed Chen-Lee-Liu equation and modulation instability in optical mono mode fibres[J]. Optical and quantum electronics, 2021, 53：286.
[16] SEADAWY A R, RIZVI S T R, MUSTAFA B, et al. Chirped periodic waves for an cubic-quintic nonlinear Schrödinger equation with self steepening and higher order nonlinearities[J]. Chaos, solitons & fractals, 2022, 156：111804.
[17] TRIKI H, BISWAS A, MILOVIC D, et al. Chirped femtosecond pulses in the higher-order nonlinear Schrödinger equation with non-Kerr nonlinear terms and cubic-quintic-septic nonlinearities[J]. Optics communications, 2016, 366：362-369.
[18] TRIKI H, PORSEZIAN K, GRELU P. Chirped soliton solutions for the generalized nonlinear Schrödinger equation with polynomial nonlinearity and non-Kerr terms of arbitrary order[J]. Journal of optics, 2016, 18：075504.
[19] SEADAWY A R, AHMED H M, RABIE W B, et al. Chirp-free optical solitons in fiber Bragg gratings with dispersive reflectivity having polynomial law of nonlinearity[J]. Optik, 2021, 225：165681.